Practice Paper

1 **a** Let X be the number of people with flu.
$X \sim B(12, 0.15)$
$P(X = 3) = 0.9078 - 0.7358$ (from tables)
$= 0.1720$

b $P(X \leq 5) = 0.9954$ (from tables)

2 **a** Not a statistic. It is not a function of solely values from the sample.
It contains a parameter μ.

b A statistic. It is a function of solely values from the sample.

3 **a** Let X be the number of bear sightings per week.
$X \sim Po(3)$
$P(X < 2) = P(X \leq 1)$
$= 0.1991$ (from tables)

b Let Y be the number of bear sightings per 2 weeks.
$Y \sim Po(6)$
$P(Y = 6) = 0.6063 - 0.4457$ (from tables)
$= 0.1606$

c From part **a** $P(X \leq 1) = 0.1991$
$P(X \leq 1 \text{ for 4 consecutive days}) = 0.1991^4$
$= 0.00157$ (3 s.f.)

4 **a** $$\mathrm{f}(x) = \begin{cases} \frac{1}{33}(ax+b) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{E}(X) = \frac{26}{11}$$

$$\mathrm{E}(X) = \int_0^\infty x\mathrm{f}(x)\,\mathrm{d}x$$

$$= \frac{1}{33}\int_1^4 (ax^2 + bx)\,\mathrm{d}x$$

$$\frac{1}{33}\left[\frac{1}{3}ax^3 + \frac{1}{2}bx^2\right]_1^4 = \frac{26}{11}$$

$$\left[\left(\frac{64}{3}a + 8b\right) - \left(\frac{1}{3}a + \frac{1}{2}b\right)\right] = 78$$

$$21a + \frac{15}{2}b = 78$$

Therefore:
$14a + 5b = 52$ as required.

4 b $\int_0^\infty f(x)\,dx = 1$

$\frac{1}{33}\int_1^4 (ax+b)\,dx = 1$

$\left[\frac{1}{2}ax^2 + bx\right]_1^4 = 33$

$\left[(8a+4b) - \left(\frac{1}{2}a + b\right)\right] = 33$

$\frac{15}{2}a + 3b = 33$ **(1)**

From part **b**

$14a + 5b = 52$ **(2)**

$5 \times$ **(1)** gives:

$\frac{75}{2}a + 15b = 165$ **(3)**

$-3 \times$ **(2)** gives:

$-42a - 15b = -156$ **(4)**

Adding **(3)** and **(4)** gives:

$-\frac{9}{2}a = 9$

$a = -2$

$b = 16$

c $F(x) = \int f(x)\,dx$

$= \frac{1}{33}\int(-2x+16)\,dx$

$= \frac{1}{33}\left(-x^2 + 16x + c\right)$

$F(1) = 0$ therefore $c = -15$

$F(x) = \frac{1}{33}\left(-x^2 + 16x - 15\right)$

At upper quartile $F(X) = 0.75$

$\frac{1}{33}\left(-x^2 + 16x - 15\right) = 0.75$

$-x^2 + 16x - 15 = 24.75$

$4x^2 - 64x + 159 = 0$

$x = \frac{64 \pm 4\sqrt{97}}{8}$

$x = 12.92\ldots$ or $x = 3.075\ldots$

since $1 \le x \le 4$

$x = 3.08$ (2 d.p) as required

d The mode occurs at the maximum of the pdf, therefore, mode = 1

5 a $L \sim \mathrm{U}[0, 80]$

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{80} & 0 \le x \le 80 \\ 0 & \text{otherwise} \end{cases}$$

b Let X be the probability that Yifan will need to leave the queue without going on the ride.

$$\mathrm{P}(X) = \frac{1}{80}(80 - 70)$$
$$= \frac{1}{8}$$

c Let Y be the probability that Yifan will go on the ride.

$$\mathrm{P}(Y) = \frac{1}{80}(80 - 65)$$
$$= \frac{15}{80}$$

Let Z be the probability that Yifan will queue for at least 30 minutes.

$$\mathrm{P}(Z) = \frac{1}{80}(80 - 30)$$
$$= \frac{50}{80}$$

Therefore:

$$\mathrm{P}(Y \mid Z) = \frac{\frac{15}{80}}{\frac{50}{80}}$$
$$= \frac{3}{10}$$

6 a Let X be the number of typing errors per page.
$X \sim \mathrm{Po}\,(4)$
Let Y be the number of typing errors per 2 pages.
$Y \sim \mathrm{Po}\,(8)$

i $\mathrm{P}(Y = 7) = 0.4530 - 0.3134$ (from tables)
$= 0.1396$

ii $\mathrm{P}(Y > 7) = 1 - \mathrm{P}(Y \le 7)$
$= 1 - 0.4530$ (from tables)
$= 0.5470$

6 b Let W be the number of errors in n pages.
$W \sim \text{Po}(4n)$
Use the approximation:
$T \sim \text{N}(4n, 4n)$
$\text{P}(W > 40) = \text{P}(T > 40.5)$ (continuity correction)
$\text{P}(T > 40.5) = 1 - \text{P}(T < 40.5) = 0.2268$

$$\text{P}(T < 40.5) = 0.7732 \Rightarrow Z = 0.7494$$
$$\text{P}(T < 40.5) = \text{P}\left(Z < \frac{40.5 - 4n}{\sqrt{4n}}\right)$$
Therefore:
$$\frac{40.5 - 4n}{\sqrt{4n}} = 0.7494$$
$$40.5 - 4n = 1.4988n^{\frac{1}{2}}$$
$$4n + 1.4988n^{\frac{1}{2}} - 40.5 = 0$$
Let $n = x^2$
$$4x^2 + 1.4988x - 40.5 = 0$$
$$x = \frac{-1.4988 \pm \sqrt{(-1.4988)^2 - 4(4)(-40.5)}}{2(4)}$$
$$= \frac{-1.4988 \pm 25.49...}{8}$$
$x = 3.000\ldots$ or $x = -3.374\ldots$
Since x must be positive $x = 3.000\ldots$
Since $n = x^2$
$n = 9$

7 a $n = 40$, $p = 0.2$
Let X be the number of seeds that grow.
$X \sim \text{B}(40, 0.2)$
Critical value is $X = 15$ (from tables)

b $\text{H}_0\!: p = 0.2$, $\text{H}_0\!: p > 0.2$
Reject H_0 – There is sufficient evidence that Benoit is correct.

c $\text{P}(X \geq 15) = 1 - 0.9971 = 0.0029$

8 a $B = 1, 2, 3, 4$ and $R = 1, 2, 3$
$X \sim (B - 1)(3 - R)$
The sample space diagram is:

		(B – 1)			
		0	**1**	**2**	**3**
(3 – R)	**2**	0	2	4	6
	1	0	1	2	3
	0	0	0	0	0

$$\text{P}(X = 4) = \frac{1}{12}$$

8 b

x	0	1	2	3	4	6
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

c Mode = 0

9 a n is large, p is small

b $H_0: p = 0.003$ and $H_1: p > 0.003$
Let X be the number of faulty bulbs.
$X \sim B(2000, 0.003)$
Use the approximation $Y \sim Po(6)$

$$\begin{aligned} P(X \geq 12) &= 1 - P(Y \leq 11) \\ &= 1 - 0.9799 \\ &= 0.0201 \end{aligned}$$

$0.0201 < 0.05$ therefore reject H_0
There is evidence that the proportion of faulty bulbs has increased.